

Introduction to Probability and Statistics - 18.05

Problem set 3

Due Friday, March 7th 2008

1. For what constant k is $f(x) = ke^{-x}$ a probability density function on $[0, 1]$ (i.e. the function is 0 outside this interval)? What is the expectation of the random variable for which this is its density function?
2. The probability density function f of a continuous random variable X is given by

$$f(x) = \begin{cases} cx + 3 & \text{for } -3 \leq x \leq 2 \\ 3 - cx & \text{for } 2 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Compute c
 - (b) Compute the distribution function of X
3. A 5 inch long pencil is broken into two pieces, where the breaking point is uniformly distributed. Let X be (the continuous) r.v. measuring the distance from the breaking point to the sharp end of the pencil. Let Y be the r.v. who is twice the distance from the breaking point to the blunt end of the pencil.
 - (a) What is the expectation and variance of X ?
 - (b) What is the expectation and variance of Y ?
 - (c) consider the events " $X = 3$ " and " $Y = 8$ ". Are these events independent?
 4. Suppose the Red Sox are playing a series of games against the Yankees until one of the teams wins three games. Further suppose that at each game (independently of the results of the other games) the Red Sox has probability 0.6 to win. Let X be the random variable "the number

of games played in the series". What is the expectation of X and what is its variance?

5. Consider the following game: I toss a coin with probability p for heads 10 times. For every toss resulting in heads I gave you one dollar and for every tails you give me two dollars. What is the expected value of the money you win/lose (as a function of p)? For which values of p will you consider participating in the game?