

Introduction to Probability and Statistics - 18.05

Problem set 4

Due Friday, March, 21st 2008

1. There are k people in a room, and 365 days in a year. Each of these k people is born on a day chosen uniformly at random from the 365 days. What is the expected number of pairs of individuals that have the same birthday?
2. A random variable has an expected value of 2 and a variance of 3. Find the expected value for the random variable $Y = 2X^2 + 5X + 4$
3. The joint probability density function of X and Y is

$$f(x, y) = 2 - x - y \text{ if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1$$

and 0 otherwise. Find the marginal density functions f_X and f_Y .

4. Suppose that two standard, fair dice are rolled and the results (X_1, X_2) are recorded. Let $Y = \max(X_1, X_2)$, $Z = \min(X_1, X_2)$ denotes the minimum and the maximum of the results, respectively.
 - (a) Find the joint probability mass function of (Y, Z) .
 - (b) Find the probability mass function of Y .
5. Let X, Y, Z be three independent uniform random variables over $[0, 1]$. Find the density function of the random variable $X + Y + Z$.

Hint: Use the density function of $X + Y$ that we computed in class.