Introduction to Probability and Statistics - 18.05

Problem set 5

Due Friday, April 4th, 2008

1. A certain bank has the policy that every transaction is rounded to the nearest dollar. So for example if the bank has to pay 11.4 dollars it actually pays only 11 while if it has to pay 4.5 it pays 5. Suppose that the bank makes 100 transactions and let X_i (for $1 \leq i \leq 100$) be the difference between the actual value of the i'th transaction and the actual amount of money transferred. I.e., in the first example above X_i would be 0.4 and in the second X_i would be -0.5. Suppose that $X_1, \ldots, X_{100}$ are independent and uniformly distributed over $(-0.5, 0.5)$.

   (a) Use Chebyshev's inequality to compute an upper bound on the probability that in these 100 transactions, there is a difference of more than 10 dollars between the actual value of all the transactions together and the total amount that was actually transferred. I.e. give a bound for $P(|X_1 + X_2 + ...X_{100}| > 10)$.

   (b) What can you say about $\frac{1}{n}\Sigma_{i=1}^{n}|X_i|$ as n becomes large, applying the law of large numbers?

2. There are 100 lightbulbs whose lifetimes are independent exponential r.v.'s with mean 5 (hours). If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one. Use the central limit theorem to approximate the probability that there is still a working bulb after 525 hours.

3. A forgetful student is always having trouble remembering where his belongings are: socks, shoes, clothes, watches, pens, notes, etc. Let X be the total number of items that he remembers their locations. Suppose he remembers the items' locations independently (with the same probability for each specific item), and we know that $E[X] =$

36. Use Chernoff to give an upper bound on the probability that he remembers more than 48 items.

4. Assume you are playing a game against a friend and your probability to win is 2/3 and to lose 1/3. Further suppose that the probability to win is independent of other games you play. Use both Chernoff bounds and Chebyshev's inequality to bound the probability that you win less than half the games when you play 30, 100, and 10^6 games.