Introduction to Probability and Statistics - 18.05

Problem set 7

Due Friday, April 25th, 2008

1. Suppose $y_1, y_2 ... y_n$ is a dataset that is a realization of a random sample from a geometric distribution with $p = \frac{1}{M}$. Determine the maximum likelihood estimate for M.

2. Suppose $x_1, \ldots, x_n$ is a realization of a uniform random variable over the interval (α, β). What are the maximum likelihood estimates for α and β?

3. Suppose we have a dataset that is a realization of a random sample Z_1, Z_2, $Z_3 ... Z_n$. Each r.v. is a uniform distribution on $[-\delta, \delta]$, with δ unknown.

   (a) Show that $X = \frac{3}{n}\Sigma_{i=1}^{n}(Z_i)^2$ is an unbiased estimator for δ^2

   (b) Is $\sqrt{X}$ biased for δ? If yes, is it postively or negatively biased?

4. Suppose we have a distribution with mean μ. Let $\bar{Z_n}$ and $\bar{W_m}$ be the sample means of two independent random samples from this distribution. Let $Y = s\bar{Z_n} + (1 - s)\bar{W_m}$, where $0 \leq s \leq 1$

   (a) Is Y biased for μ? Why or why not?

   (b) When is Y most efficient?

5. $X_1, \ldots, X_n$ is a random sample from an exponential distribution with parameter $\lambda > 0$. For every real number c, consider the estimator $T_c = c \cdot (X_1 + X_2 + \cdots + X_n)$.

   (a) Which value of c yields an unbiased estimator for $1/\lambda$?

   (b) Which value of c yields the most efficient estimator for $1/\lambda$?