

Introduction to Probability and Statistics - 18.05

Test 2

Wednesday April 16, 2008

Guidelines: You have 55 minutes to complete the test. You are allowed to use calculators but no written material is allowed. Your answers must include **clear and short** explanations. Correct answers with poor explanations will not receive full credit. The credit points are divided evenly between the 4 questions. Don't forget to write your full name at the top of the answers sheet. Good luck!

1. Scientists are trying to estimate the average number of eggs in a red-footed falcon's nest. They do that by sampling nests independently at random, counting the number of eggs in each sampled nest and taking their estimate to be the average of the samples. While they do not know the average, they do know that the variance is at most 3. How many nests do they have to sample so that they can be sure that with probability at least $999/1000$, their estimated average will be at most 2 away from the real average?
2. (a) What is linearity of expectation? Under which conditions on the random variables it holds?
(b) Suppose the scientists from the previous question sample n nests. From each nest they also take one egg to make some measurements in the lab. Once they are done, they return the eggs to the nests (one egg to each nest). However, since they all look alike, each egg has equal probability to be placed in each one of the n nests. What is the expected number of eggs to be placed back in their original nests?
3. (a) State formally the Central Limit Theorem.
(b) The time intervals (in years) between eruptions of a certain volcano are exponentially distributed with $\lambda = 1/100$. Assume that the lengths of the intervals between different eruptions are independent. Use the Central Limit Theorem to estimate the probability that the 10th eruption (from today) will happen after more than 1200 years.
4. Let X be a uniform random variable over the interval $[0, 10]$ and let Y be a uniform random variable over the interval $[X, 10]$.
 - (a) Are X and Y independent?
 - (b) What is the joint density function of X and Y ?
 - (c) What is the probability that $Y > 5$?