Introduction to Probability and Statistics - 18.05

Test 3

1. (a) Define formally the notion of mean squared error.

   (b) Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with parameter λ. Consider $\frac{1}{\bar{X}_n}$ as an estimator for λ (where $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$). Is it an unbiased estimator? If not, is it positively or negatively biased?

2. Let $x_1, x_2 \ldots x_n$ be a dataset that is a realization of a random sample from a geometric distribution with parameter p.

   (a) What is the likelihood function $L(p)$?

   (b) What is the loglikelihood function $\ell(p)$?

   (c) Determine the maximum likelihood estimate for p.

3. (a) Explain why type II error in hypotheses testing can be as close as we want to $1 - \alpha$, where α is the type I error.

   (b) $X_1, \ldots, X_{10}$ is a random sample from a uniform distribution over the interval (δ_1, δ_2) (where $\delta_1 < \delta_2$). You want to test the hypothesis $H_0 : \delta_1 = -\delta_2$ against $H_1 : \delta_1 > -\delta_2$. You do that by counting the number of samples that are positive. What is the critical region for the number of positive samples with respect to significance level 0.05?

4. The interval $(1.6, 7.8)$ is a 95% confidence interval for the parameter μ of a normal distribution with μ and σ^2 (both unknown). The interval is based on 16 observations, constructed according to the studentized mean procedure.

   (a) What is the mean of the (unknown) dataset?

   (b) Please construct a 99% confidence interval for μ.