

## Introduction to Probability and Statistics - 18.05 Spring 2008

### Problem Set 6 Solutions

1. Random sample is defined as sequence of independent and identical random variables. Independence implies that  $Var[X_1 + X_2] = Var[X_1] + Var[X_2]$ , Therefore, Let

$$\begin{aligned} X &= X_1 + X_2 + X_3 \\ E[X^2] &= (E[X])^2 + Var[X] \\ &= (E[X_1 + X_2 + X_3])^2 + Var[X_1 + X_2 + X_3] \\ &= (E[X_1] + E[X_2] + E[X_3])^2 + Var[X_1] + Var[X_2] + Var[X_3] \\ &= (2 + 2 + 2)^2 + 3 \\ &= 39 \end{aligned}$$

2. The r.v. which is the number of wheat bread brands has Binomial distribution with parameters 7 and  $p$  (which is unknown). So the statistical model will contain all the Binomial distributions with these parameters.
3. (a) The average is 1 if and only if it is 1 on all four draws,  $(\frac{1}{4})^4 = \frac{1}{256}$   
(b) Let's compute the probability that the maximal is not 6, which means the number we draw each time can be any of 1, 3 or 4.

$$\begin{aligned} P[MAX = 6] &= 1 - P[MAX \neq 6] \\ &= 1 - \left(\frac{3}{4}\right)^4 \\ &= \frac{175}{256} \end{aligned}$$

- (c) We have to choose the two (out of four) draws that are less than 2, the probability that this is indeed the case is  $(\frac{1}{4})^2$  (i.e. both

need to be 1), and the probability that the other two are more than 2 (i.e. 3,4 or 6) is  $(\frac{3}{4})^2$ . So we get:

$$\binom{4}{2} \times (\frac{1}{4})^2 (\frac{3}{4})^2 = \frac{54}{256}$$

4. We have  $\bar{X}_n = 0$ . The sample standard deviation is:

$$\begin{aligned} S_n &= \sqrt{\frac{1}{2N} \sum_{i=-N}^N (i - \bar{X}_n)^2} \\ &= \sqrt{\frac{2}{2N} \times (1^2 + 2^2 + 3^2 \dots + N^2)} \\ &= \sqrt{\frac{N(N+1)(2N+1)}{6N}} \\ &= \sqrt{\frac{(N+1)(2N+1)}{6}} \end{aligned}$$