Introduction to Probability and Statistics - 18.05

Problem set 8 solutions

1. Since mean and variation unknown and n is small, we use the t-distribution. $m = 10-1 = 9$, $\bar{x_n} = 3.3288, s_n = \sqrt{\frac{1}{n-1}\Sigma_{i=1}^{n}(x_i - \bar{x_n})^2} = 0.1562$, $\gamma = 0.9, \alpha = 0.1, \frac{\alpha}{2} = 0.05$. From the table we get, $t_{9,0.05} = 1.833$ Therefore,

$$c_l = \bar{x_n} - 1.833\frac{s_n}{\sqrt{n}} = 3.2399$$

$$c_u = \bar{x_n} + 1.833\frac{s_n}{\sqrt{n}} = 3.4197$$

The 90% confidence interval is (3.2399, 3.4197).

2. Let $z_{0.05}\frac{\sigma}{\sqrt{n}} = 0.03$ where $\sigma^2 = 0.14$ and $z_{0.05} = 1.645$
$\Rightarrow n = 421$.

3. Having $\bar{x_n} = 3.3288, z_{0.05} = 1.645, \sigma^2 = 0.14, n = 10$, we get

$$c_l = \bar{x_n} - z_{0.05}\frac{\sigma}{\sqrt{n}} = 3.1341$$

$$c_u = \bar{x_n} + z_{0.05}\frac{\sigma}{\sqrt{n}} = 3.5234$$

4. Use Chebyshev inequality: For 90% confidence interval, take $\epsilon = \sqrt{10}\sigma$ where $\sigma^2 = 0.14$. Also we know that $t = \bar{x_n} = 3.3288$. By Chebyshev,

$$\Pr[|\bar{X_n} - \mu| > \epsilon] < \frac{\sigma^2}{\epsilon^2} = \frac{1}{10}$$

Therefore the interval $(t - \epsilon, t + \epsilon) = (2.1456, 4.5120)$.

5. Plugging the formular for β and α, respectively, we get

$$\hat{\beta} = \frac{n\Sigma_{i=1}^{n}x_iy_i - (\Sigma_{i=1}^{n}x_i)(\Sigma_{i=1}^{n}y_i)}{n\Sigma_{i=1}^{n}x_i^2 - (\Sigma_{i=1}^{n}x_i)^2} = -0.25$$

Similarly, $\alpha = 2.35 \Rightarrow y = 2.35 - 0.25x$. For the second part, add in the fourth point (0,0), we get $y = 0.957 + 0.108x$