Introduction to Probability and Statistics - 18.05

Problem set 9 solutions

1. $P[w > 5] = P[w - 3 > 5 - 3] = P[z > 2] = 1 - \Phi(2) = 0.0228$

2. $P[(\bar{Y_n})^2 > c] = 0.05$
   $\Rightarrow P[\bar{Y_n} > \sqrt{c}] + P[\bar{Y_n} < -\sqrt{c}] = 0.05$
   $\Rightarrow P[\bar{Y_n} > \sqrt{c}] = 0.025$
   $\Rightarrow \sqrt{c} = 1.96, c = 3.84$
   $\Rightarrow$ The critical region is $[3.84, \infty)$

3. Assuming H_0 then the probability of having a defect screw is 1/10. The probability of a type I error is same as the probability of getting 3, 4 or 5 defect screws in a sample of 5:

   $$\binom{5}{3}(\frac{1}{10})^3(\frac{9}{10})^2 + \binom{5}{4}(\frac{1}{10})^4(\frac{9}{10})^1 + \binom{5}{5}(\frac{1}{10})^5(\frac{9}{10})^0 = 0.00856$$

   Similarly in the second situation, the probability of getting a defect screw is $\frac{20}{100} = 0.2$, accordingly we want to compute the probability of getting 0 or 1 or 2 defect screws:

   $$\binom{5}{0}(0.2)^0(0.8)^5 + \binom{5}{1}(0.2)^1(0.8)^4 + \binom{5}{2}(0.2)^2(0.8)^3 = 0.942$$

4. Let $Z = X_1 + X_2$. Assuming H_0, then the density function of Z is: $f(z) = z$ for $0 \leq z \leq 1$, $f(z) = 2 - z$ for $1 < z \leq 2$ and 0 otherwise. The expectation of Z is 1. Therefore, values close to 1 support H_0, while values smaller than 1 support H_1. We need to find a number c such that $\int_0^c f(z)dz = 0.05$. $\int_0^c f(z)dz = \frac{z^2}{2}|_0^c = \frac{c^2}{2} = 0.05$. This gives, $c = \frac{\sqrt{10}}{10} = 0.316$.